Geodynamics

Brittle deformation and faulting
Lecture 11.6 - Predicting fault orientations

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Goals of this lecture

• Introduce Anderson’s theory and how it can be applied to predict fault orientations
Faulting

Beartooth Plateau, Wyoming, USA
Faulting

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Anderson’s theory

- Anderson formulated the Mohr-Coulomb criterion in terms of principal stresses and lithostatic pressure \((\sigma_{yy} = \rho g y)\)

- Anderson assumed \(\sigma_{yy} = \sigma_3\) for a reverse fault, \(\sigma_{yy} = \sigma_1\) for a normal fault and \(\sigma_{yy} = \sigma_2 = (\sigma_1 + \sigma_3)/2\) for a strike-slip fault

Fig. 5.8, Stüwe, 2007
Anderson’s theory

- In terms of differential stress, Anderson’s theory is

Reverse: \[ \sigma_d = \sigma_1 - \sigma_3 = \frac{2 \left( c + f_s (\sigma_{yy} - \rho_w g y) \right)}{\sqrt{f_s^2 + 1 - f_s}} \]

Normal: \[ \sigma_d = \sigma_1 - \sigma_3 = \frac{-2 \left( c - f_s (\sigma_{yy} - \rho_w g y) \right)}{\sqrt{f_s^2 + 1 + f_s}} \]

Strike-slip: \[ \sigma_d = \sigma_1 - \sigma_3 = \frac{2 \left( c + f_s (\sigma_{yy} - \rho_w g y) \right)}{\sqrt{f_s^2 + 1}} \]
Now we’ll look at how to apply Anderson’s theory to predict the dip angle $\beta$ of normal and reverse faults.

Assume principal stresses

$$\sigma_{yy} = \rho gy$$
$$\sigma_{xx} = \rho gy + \Delta \sigma_{xx}$$

where $\Delta \sigma_{xx}$ is the tectonic deviatoric stress, which is positive for a reverse fault and negative for a normal fault.
Predicting fault orientation

- We first need to relate \( \sigma_{xx} \) and \( \sigma_{yy} \) to \( \sigma_n \) and \( \sigma_s \) in order to apply Amonton’s law by using the equation for the normal and shear stresses in a coordinate system rotated by angle \( \theta \) with respect to the principal stresses (see Lecture set 3)

\[
\sigma_n = \frac{1}{2} (\sigma_{xx} + \sigma_{yy}) + \frac{1}{2} (\sigma_{xx} - \sigma_{yy}) \cos 2\theta
\]

\[
\sigma_s = -\frac{1}{2} (\sigma_{xx} - \sigma_{yy}) \sin 2\theta
\]

- Note that here, \( \theta \) is with respect to vertical, \( \theta = \pi/2 - \beta \)
Predicting fault orientation

If we plug in the values for \( \sigma_{xx} \) and \( \sigma_{yy} \), we find

\[
\sigma_n = \rho gy + \frac{\Delta \sigma_{xx}}{2} (1 + \cos 2\theta)
\]

\[
\sigma_s = -\frac{\Delta \sigma_{xx}}{2} \sin 2\theta
\]

Inserting the values above into the form of Amonton’s law that includes pore fluid pressure, \( |\tau| = f_s (\sigma_n - p_w) \), yields

\[
\pm \frac{\Delta \sigma_{xx}}{2} \sin 2\theta = f_s \left[ \rho gy - p_w + \frac{\Delta \sigma_{xx}}{2} (1 + \cos 2\theta) \right]
\]

Note that the upper sign is for reverse faults (\( \Delta \sigma_{xx} > 0 \)) and the lower for normal faults (\( \Delta \sigma_{xx} < 0 \)).
Predicting fault orientation

Let $\Delta \sigma_{xx}$ refer to the difference in stress caused by tectonic forces. The expression for $\Delta \sigma_{xx}$ can be rearranged to solve for it:

$$\Delta \sigma_{xx} = \frac{2f_s(\rho g y - p_w)}{\pm \sin 2\theta - f_s(1 + \cos 2\theta)}$$

- If we assume that faulting will occur with the minimum tectonic stress, then $|\Delta \sigma_{xx}|$ should be minimized.

- By setting $d\Delta \sigma_{xx}/d\theta = 0$, we find:

  $$\tan 2\theta = \mp \frac{1}{f_s} \quad \text{or} \quad \tan 2\beta = \pm \frac{1}{f_s}$$

where the upper sign again applies to reverse faults and the lower to normal faults.
Finally, the two equations from the previous slide can be combined to yield the tectonic stresses corresponding to angle $\theta$ or $\beta$

$$\Delta \sigma_{xx} = \pm f_s (\rho gy - p_w) \left(1 + f_s^2\right)^{-1/2} \mp f_s$$

where the upper sign again corresponds to reverse faults and the lower to normal faults.
Let’s see what you’ve learned…

• If you’re watching this lecture in Moodle, you will now be automatically directed to the quiz!