Geodynamics

Plate-driving forces
Lecture 10.3 - Onset of thermal convection

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Goal of this lecture

- Introduce the Rayleigh number as a means for estimating the conditions under which mantle convection will occur.
Onset of thermal convection

- For a fluid heated to temperature $T_1$ at its base and cooled to temperature $T_0$ at its upper surface thermal buoyancy will drive convection if the viscous resistance to fluid flow is overcome.

- When the temperature difference $T_1 - T_0$ is small, convection will not occur and the fluid velocities will be $u = v = 0$, and temperature will not change with time ($\partial / \partial t = 0$) or along the $x$ axis ($\partial / \partial x = 0$), reducing the heat transfer equation to

$$\frac{d^2 T_c}{dy^2} = 0$$

where $c$ indicates conductive heat transfer.

Fig. 6.38, Turcotte and Schubert, 2014
Onset of thermal convection

- For the thermal boundary conditions $T = T_0$ at $y = -b/2$ and $T = T_1$ at $y = b/2$, the solution to the heat transfer equation is

$$T_c = \frac{T_1 + T_0}{2} + \frac{(T_1 - T_0)}{b} y$$

where $b$ is the thickness of the fluid layer.

- Convection will begin when the temperature $T$ just begins to exceed the conductive temperature $T_c$.

- In this case, the temperature difference $T'$ is extremely small:

$$T' \equiv T - T_c$$

Fig. 6.38, Turcotte and Schubert, 2014
Onset of thermal convection

- The temperature difference $T'$ can be inserted into the heat transfer and force balance equations for a 2D incompressible viscous fluid to determine the conditions under which convection will occur.

- From this, a dimensionless number known as the Rayleigh number can be defined:

$$Ra = \frac{\rho_0 g \alpha_v (T_1 - T_0) b^3}{\eta \kappa}$$

![Diagram showing the onset of thermal convection with labels $T_0$, $T_1$, $b$, and $\lambda = 2\sqrt{2}b$.](image)

Fig. 6.38, Turcotte and Schubert, 2014
Onset of thermal convection

• Instability and convection will occur when

\[ Ra > \left( \frac{\pi^2 + \frac{4\pi^2 b^2}{\lambda^2}}{\frac{4\pi^2 b^2}{\lambda^2}} \right)^3 \]

where \( \lambda \) is the wavelength of convection

• The flow is thus stable when \( Ra \) is less than the right side of the equation above

• The critical Rayleigh number when convection begins is thus

\[ Ra \equiv Ra_{cr} = \left( \frac{\pi^2 + \frac{4\pi^2 b^2}{\lambda^2}}{\frac{4\pi^2 b^2}{\lambda^2}} \right)^3 \]

Minimum at \( \lambda = 2\sqrt{2b} \)

Fig. 6.39, Turcotte and Schubert, 2014
For a fluid cooled from above, heated from within and with no heat flux across its base the Rayleigh number equation is

$$ Ra_H = \frac{\alpha_v \rho_0^2 g H b^5}{k \eta \kappa} $$

where $H$ is the heat production per unit mass and $k$ is the thermal conductivity.

The equation above is appropriate for the Earth’s mantle.
Onset of thermal convection

\[ Ra_H = \frac{\alpha_v \rho_0^2 g H b^5}{k \eta \kappa} \]

- The critical Rayleigh number for an internally heated fluid is \( Ra_{cr} = 2772 \)
- What is the Rayleigh number for the upper mantle?
  - Should it convect?
  - What about the whole mantle?

Assume \( \alpha_v = 3 \times 10^{-5} \, \text{K}^{-1}, \rho_0 = 4000 \, \text{kg m}^{-3}, H = 9 \times 10^{-12} \, \text{W kg}^{-1}, k = 4 \, \text{W m}^{-1} \, \text{K}^{-1}, \eta = 1 \times 10^{21} \, \text{Pa s}, \) and \( \kappa = 1 \, \text{mm}^2 \, \text{s}^{-1} \)
Onset of thermal convection

\[ \text{Ra}_H = \frac{\alpha_v \rho_0^2 g H b^5}{k \eta \kappa} \]

- The critical Rayleigh number for an internally heated fluid is \( \text{Ra}_{\text{cr}} = 2772 \)
- What is the Rayleigh number for the upper mantle? \( 1.3 \times 10^6 \)
  - Should it convect? Yes
- What about the whole mantle? \( 1.76 \times 10^9 \)
- Assume \( \alpha_v = 3 \times 10^{-5} \text{ K}^{-1}, \rho_0 = 4000 \text{ kg m}^{-3}, H = 9 \times 10^{-12} \text{ W kg}^{-1}, k = 4 \text{ W m}^{-1} \text{ K}^{-1}, \eta = 1 \times 10^{21} \text{ Pa s}, \text{ and } \kappa = 1 \text{ mm}^2 \text{ s}^{-1} \)
Let’s see what you’ve learned…

- If you’re watching this lecture in Moodle, you will now be automatically directed to the quiz!