Geodynamics

Plate-driving forces
Lecture 10.2 - Physics of thermal convection

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Goals of this lecture

• Introduce basic equations for the **force balance in a viscous fluid in 2D**

• Present the corresponding **2D heat flow equation**
Thermal convection

• Thermal convection in the Earth results from **buoyancy forces** owing to **thermal expansion** of mantle rocks.

• For an incompressible viscous fluid, the *vertical* force balance of pressure, gravity and viscous forces in 2D is

\[
0 = -\frac{\partial p}{\partial y} + \rho g + \eta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]

where \( p \) is pressure, \( \rho \) is density, \( \eta \) is viscosity and \( v \) is velocity in the \( y \) direction.

• To account for the buoyancy forces from thermal expansion, the density term must be variable

\[
\rho = \rho_0 + \rho'
\]

where \( \rho_0 \) is a reference density and \( \rho' \) is a density perturbation much smaller than \( \rho_0 \).
Thermal convection

- If the variable density is substituted into the 2D force balance equations and the hydrostatic pressure based on the reference density is eliminated by using $P = p - \rho_0 gy$ the horizontal and vertical force balance equations become

\[ 0 = -\frac{\partial P}{\partial x} + \eta \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  
\[ 0 = -\frac{\partial P}{\partial y} + \rho' g + \eta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]
Thermal convection

- A change in density as a result of thermal expansion is
  \[ \rho' = -\rho_0 \alpha_v (T - T_0) \]
  where \( \alpha_v \) is the volumetric coefficient of thermal expansion and \( T_0 \) is a reference temperature corresponding to the reference density \( \rho_0 \).

- The vertical force balance including thermal buoyancy is thus
  \[ 0 = -\frac{\partial P}{\partial y} + \eta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - g \rho_0 \alpha_v (T - T_0) \]
Thermal convection

- In order to determine the thermal buoyancy, we require a 2D equation for heat transfer via conduction and convection.
- Turcotte and Schubert give a detailed derivation, arriving at the following relationship for heat conduction and convection in 2D:

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

where \( u \) is the velocity in the \( x \) direction and \( \kappa \) is the thermal diffusivity.
Thermal convection

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Time dependence

- Convection
- Conduction

where \( u \) is the velocity in the \( x \) direction and \( \kappa \) is the thermal diffusivity.
Let’s see what you’ve learned…

- If you’re watching this lecture in Moodle, you will now be automatically directed to the quiz!