Geodynamics

Basics of fluid mechanics
Lecture 9.6 - Pipe flow

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Goals of this lecture

• Calculate the flow velocity in pipe flow

• Introduce the **Reynolds number** and its meaning
Pipe flow
Flow in a pipe

• In contrast to our 1D channel flows earlier, in which the channel was of infinite width, in pipe flow we consider fluid flowing in a round pipe of radius $R$.

• As you can imagine, in this case fluid flow is driven solely by a pressure difference along the length of the pipe.
Flow in a pipe

- The pressure difference \((p_1 - p_0)\) acts along a pipe section of length \(l\), and we assume the velocity varies only as a function of the distance from the center of the pipe \(r\).

- For a given portion of the pipe of radius \(r\), the pressure force on the ends on the cylindrical volume will be balanced by a shear force acting on it.
Flow in a pipe

• Thus, we can set the pressure force equal to the shear force to balance the forces

\[(p_1 - p_0)\pi r^2 = -2\pi rl\tau\]

Pressure force \quad Shear force

• This relationship can be rewritten more simply as

\[\tau = \frac{r}{2} \frac{dp}{dx}\]
Flow in a pipe

• For a Newtonian fluid in a cylinder, the constitutive relationship is of the form

\[ \tau = \eta \frac{du}{dr} \]

• This flow law can be inserted into the force balance from the previous slide to give an equation for the slope of the velocity in terms of pressure

\[ \frac{du}{dr} = \frac{r}{2\eta} \frac{dp}{dx} \]
Flow in a pipe

- One integration give the equation for the velocity of flow of fluid in a pipe

\[ u = -\frac{1}{4\eta} \frac{dp}{dx} (R^2 - r^2) \]

- Based on this equation, what is the maximum flow velocity in the pipe? Note, you do not need to solve for values, simply find this symbolically.
Flow in a pipe

Based on this equation, what is the maximum flow velocity in the pipe? Note, you do not need to solve for values, simply find this symbolically.

The maximum flow velocity occurs at \( r = 0 \), so

\[
\dot{u}_{\text{max}} = -\frac{R^2}{4\eta} \frac{dp}{dx}
\]
Laminar flow and the Reynolds number

- Our pipe flow velocity calculation will work properly, provided the flow of the fluid in the pipe is laminar, and not turbulent.
- Laminar flow will occur in parallel flow lines, whereas eddies will form in turbulent flows.
- The determination of the flow type can be made by calculating the dimensionless Reynolds number:

\[
Re = \frac{\rho \bar{u} D}{\eta}
\]

where \( \rho \) is the fluid density, \( \bar{u} \) is the mean flow velocity in the pipe and \( D \) is the pipe diameter (2\( R \)).
- Flow becomes turbulent in the pipe for Reynolds numbers greater than ~2200.
Let’s see what you’ve learned…

• If you’re watching this lecture in Moodle, you will now be automatically directed to the quiz!