Basics of elasticity
Lecture 5.6 - Isotropic stress

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Goals of this lecture

- Present the equations for the volumetric change in rock under isotropic stress
Isotropic stress

• If all three principal stresses are equal, \( \sigma_1 = \sigma_2 = \sigma_3 = p \) and the state of stress is isotropic

• In this case, the principal strains are also equal, \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \frac{1}{3}\Delta \)
Isotropic stress

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• From the equations of elasticity in terms of stress we find:

\[
p = \left( \frac{3\lambda + 2G}{3} \right) \Delta \equiv K\Delta \equiv \frac{1}{\beta} \Delta
\]

where \( K \) is the bulk modulus and its reciprocal \( \beta \) is the compressibility.
Isotropic stress

- Any change in volume of rock must conserve mass
- For a parcel of rock with volume $V$ a change in volume $\delta V$ will result in a change in density $\delta \rho$, or

$$\delta(\rho V) = 0$$
Isotropic stress

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- For a parcel of rock with volume $V$ a change in volume $\delta V$ will result in a change in density $\delta \rho$, or
  \[ \delta (\rho V) = 0 \]
- In terms of dilatation of the rock parcel $\Delta$ we can say
  \[ \frac{-\delta V}{V} = \Delta = \frac{\delta \rho}{\rho} \]
  assuming $\Delta$ is small
- Thus, the density change as a function of pressure, compressibility and initial density is simply
  \[ \delta \rho = \rho \beta p \]
Isotropic stress

- Finally, both the \textbf{bulk modulus} and \textbf{compressibility} can be found from the elastic properties of rock

\[
K = \frac{1}{\beta} = \frac{E}{3(1 - 2\nu)}
\]
Isotropic stress

- Finally, both the bulk modulus and compressibility can be found from the elastic properties of rock

\[ K = \frac{1}{\beta} = \frac{E}{3(1 - 2\nu)} \]

- What does this suggest about rock (in)compressibility as a function of Poisson’s ratio \( \nu \)?

- What happens to the bulk modulus \( K \) as \( \nu \) approaches 1/2?
Let’s see what you’ve learned…

• If you’re watching this lecture in Moodle, you will now be automatically directed to the quiz!