Introduction to Quantitative Geology
Lecture 7
Advection of the Earth’s surface:
Fluvial incision and rock uplift

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30.3.2015
Exercise 3

Any questions or problems?
Mathematics concept review: Derivatives

- As we saw with diffusive hillslopes, the equation for the elevation of the hillslopes has the general form of an inverted parabola.

- For a hillslope of this form, the following values can be calculated:

  Elevation: \( h(x) = -x^2 + 1 \)

  Gradient (slope):

  Curvature:
Mathematics concept review: Derivatives

- As we saw with diffusive hillslopes, the equation for the elevation of the hillslopes has the general form of an **inverted parabola**
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  **Elevation:** \( h(x) = -x^2 + 1 \)

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  **Curvature:**
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For a hillslope of this form, the following values can be calculated

**Elevation:** $h(x) = -x^2 + 1$

**Gradient (slope):** $h'(x) = \frac{\partial h}{\partial x} = -2x$

**Curvature:** $h''(x) = \frac{\partial^2 h}{dx^2} = -2$
Python concept review: Arrays

• An array is a data structure with a specified number of ‘slots’ or places that can contain data

```python
# Define an array
>>> x = np.linspace(0.0, 20.0, 11)
```

```python
# Define another array
>>> y = np.arange(0.0, 20.0, 1.0)
```

```python
# Define a range
>>> range(5)
[0, 1, 2, 3, 4]  # 5 items in list!
```

### Array x
```
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
```

### Array indicies
```
0
1
2
3
4
5
6
7
8
9
```

### Array y
```
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
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### Array indicies
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Python concept review: Arrays

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### Define another array

```python
>>> y = np.arange(0.0, 20.0, 2.0)
```

### Define a range

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>>> range(5)
[0, 1, 2, 3, 4]
```

### Define the same range

```python
>>> range(0, 5, 1)
[0, 1, 2, 3, 4]
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### Python concept review: Arrays

- **Array** is a data structure with a specified number of ‘slots’ or places that can contain data.

#### Examples

Define an array:

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>>> x = np.linspace(0.0, 20.0, 11)
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#### Array y and indicies:

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**LAST ITEM OMITTED**
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>>> range(0, 5, 1)
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## An array is a data structure with a specified number of ‘slots’ or places that can contain data
Python concept review: *for* loops

# Define a for loop

```python
>>> for increment in range(5):
...     print(increment)
...
0
1
2
3
4
```

- A *for* loop will *repeat a section of code for each value of the given range*.
Python concept review: for loops

# Define a for loop
>>> for increment in range(5):
...     print(increment)
... 0 1 2 3 4

start for loop
increment = 0
print increment
0 written to screen

increment = 1
print increment
1 written to screen

... 
increment = 4
print increment
4 written to screen

hit end of range(5)
exit for loop

- A for loop will repeat a section of code for each value of the given range
Python concept review: for loops

# Define a for loop
>>> for increment in range(5):
    ...    print(increment)
... 0
1
2
3
4

# Define another loop
>>> for increment in range(len(x)):
    ...    print(x[increment], increment)
... (0.0, 0)
(2.0, 1)
(4.0, 2)
... (20.0, 10)

• A for loop will repeat a section of code for each value of the given range

start for loop
increment = 0
print increment
0 written to screen
increment = 1
print increment
1 written to screen
...
increment = 4
print increment
4 written to screen
hit end of range(5)
exit for loop
Goals of this lecture

• Introduce the advection equation

• Discuss application of the advection equation to bedrock river erosion

• Introduce advective heat transfer
What is advection?

- Advection involves a lateral translation of some quantity.
- For example, the transfer of heat by physical movement of molecules or atoms within a material. A type of convection, mostly applied to heat transfer in solid materials.

http://homepage.usask.ca/~sab248/
What is advection?

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Diffusion equation

- Last week we were introduced to the diffusion equation

\[ q = -\rho \kappa \frac{\partial h}{\partial x} \]

- Flux (transport of mass or transfer of energy) proportional to a gradient

\[ \frac{\partial h}{\partial t} = -\frac{1}{\rho} \frac{\partial q}{\partial x} \]

- Conservation of mass: Any change in flux results in a change in mass/energy
Diffusion equation

\[ \frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2} \]

- Substitute the upper equation on the left into the lower to get the classic **diffusion equation**

- \( q = \text{sediment flux per unit length} \)
- \( \rho = \text{bulk sediment density} \)
- \( \kappa = \text{sediment diffusivity} \)
- \( h = \text{elevation} \)
- \( x = \text{distance from divide} \)
- \( t = \text{time} \)
Advection and diffusion equations

- This week we meet the **advection equation**

\[
\begin{align*}
\text{Diffusion} \quad \frac{\partial h}{\partial t} &= -\kappa \frac{\partial^2 h}{\partial x^2} \\
\text{Advection} \quad \frac{\partial h}{\partial t} &= c \frac{\partial h}{\partial x}
\end{align*}
\]
Advection and diffusion equations

This week we meet the advection equation

Two key differences:

- Change in mass/energy with time proportional to gradient, rather than curvature (or change in gradient)

- **Advection coefficient** $c$ has units of $[L/T]$, rather than $[L^2/T]$
Advection and diffusion equations

River channel profiles

• This week we meet the advection equation
• Two key differences:
  • Change in mass/energy with time proportional to gradient, rather than curvature (or change in gradient)
  • Advection coefficient $c$ has units of $[L/T]$, rather than $[L^2/T]$
Advection and diffusion equations

River channel profiles

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\[ \frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x} \]

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**Diffusion**

$$\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}$$

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Diffusion

Advection

Fig. 1.7, Pelletier, 2008
Advection and diffusion equations

River channel profiles

- **Advection**
  - Rate of erosion is directly proportional to hillslope gradient (curvature)
  - Also, no conservation of mass (deposition)

- **Diffusion**
  - Rate of erosion depends on change in hillslope gradient (curvature)

**Advection**

\[ \frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2} \]

**Diffusion**

\[ \frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x} \]

---

Fig. 1.7, Pelletier, 2008
Advection at a constant rate \( c \)

- Surface elevation changes in direct proportion to surface slope.
- Result is lateral propagation of the topography or river channel profile.
- Although this is interesting, it is not that common in nature.

\[
\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}
\]
Advection of the Earth’s surface: Two cases

- **Bedrock river erosion**
  - Purely an advection problem with a spatially variable advection coefficient

- **Advective heat transport**
  - Combination of advection and diffusion
Bedrock river erosion

- Not much bedrock being eroded here…
Bedrock river erosion

- Rapid bedrock incision has formed a steep gorge in this case

Kali Gandaki river gorge, central Nepal
http://en.wikipedia.org/
River erosion as an advection process

- With a constant advection coefficient $c$, we predict lateral migration of the river profile at a constant rate ($c$)
River erosion as an advection process

- With a constant advection coefficient $c$, we predict lateral migration of the river profile at a constant rate $(c)$
- Do you think this works in real (bedrock) rivers?
River erosion as an advection process

- With a constant advection coefficient \( c \), we predict lateral migration of the river profile at a constant rate \( c \).
- Do you think this works in real (bedrock) rivers?
- What might affect the rate of lateral migration?
What affects the efficiency of river erosion?

- The **amount of water flowing** in the river (discharge) and sediment
- The **slope** of the river channel
- The **strength of the underlying bedrock**
What affects the efficiency of river erosion?

- The **amount of water flowing** in the river (discharge) and sediment
- The **slope** of the river channel
- The **strength** of the underlying bedrock

- **Are these constant?**
Stream-power model of river incision

- Rather than being constant, the rate of lateral advection in river systems is **spatially variable**

\[
\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}
\]

where \(k_f\) is a material property of the bedrock (erodibility), \(w\) is the channel width, and \(Q\) is discharge
Stream-power model of river incision

Rather than being constant, the rate of lateral advection in river systems is spatially variable

\[ \frac{\partial h}{\partial t} = k_f \frac{h}{w} Q \frac{\partial h}{\partial x} \]

where \( k_f \) is a material property of the bedrock (erodibility), \( w \) is the channel width, and \( Q \) is discharge

This is known as the stream-power erosion model
Stream-power model of river incision

- If we assume precipitation is uniform in the drainage basin, discharge $Q$ will scale with drainage basin area, so we can modify our equation to read

$$\frac{\partial h}{\partial t} = \frac{k_f}{w} Q \frac{\partial h}{\partial x} \quad \rightarrow \quad \frac{\partial h}{\partial t} = K A^m S^n$$

where $K$ is an erosional efficiency factor (accounts for lithology, climate, channel geometry, sediment supply, etc. (!)), $A$ is upstream drainage area, $S$ is channel slope, and $m$ and $n$ are area and slope exponents.

- If we assume the drainage basin area increases with distance from the drainage divide $x$, we can replace the area with an estimate $A = x^{5/3}$.
Test your might

\[ \frac{\partial h}{\partial t} = U - KA^mS^n \]

1. Based on our stream-power erosion equation, what general form would a channel profile take?

2. If we assume we have reached a steady state (\( \frac{\partial h}{\partial t} = 0 \)) and \( n = 1 \), erosion must balance uplift \( U \) everywhere.

3. If we further assume precipitation is constant, bedrock erodibility is constant and \( A = x^{5/3} \), how would the channel steepness vary as you move downstream from the divide?

4. Think about how \( S \) would change as \( x \) increases.

Initial geometry

\[ h \]

\[ U \uparrow \]

\[ x \]
Evolution of a channel profile

Fig. 3.23, Allen, 1997

- A few stream-power erosion observations:
  - Stream power increases downstream as the discharge grows
  - Steeper slopes occur upstream where the discharge is low
  - Incision migrates upstream until a balance is attained between erosion and uplift
4.29 Thermal Structure of the Subducted Lithosphere

The subduction of the cold oceanic lithosphere into the deep mantle is a primary mechanism for the transport of heat from the interior of the Earth to its surface. Hot mantle rock comes to the surface at accretional plate boundaries (ocean ridges) and is cooled by heat loss to the seafloor. The result is a cold thermal “boundary layer,” the oceanic lithosphere. The thermal structure of this boundary layer was determined in Sections 4–16 and 4–17.

The cold subducted lithosphere is gradually heated and eventually becomes part of the convecting mantle. Upward convective heat transfer through the mantle involves the sinking of cold thermal anomalies (descending lithosphere at ocean trenches) and the rising of hot thermal anomalies (mantle plumes). The density differences associated with the lateral temperature variations provide the driving force for the mantle convective circulation.

In this section we discuss the temperature distribution in the subducted oceanic lithosphere.

Isotherms in a lithosphere descending at an angle of 45° into the mantle

- Advection is important in tectonically active settings

Figure 4.58, Turcotte and Schubert, 2002
4.29 Thermal Structure of the Subducted Lithosphere

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*Advection is important in tectonically active settings*
Advective heat transfer

What happens when a parcel of rock is advected?

Fig. 3.13, Stüwe, 2007
Advective heat transfer

- What happens when a parcel of rock is advected?
- The rock carries with it thermal energy (heat)
Advective heat transfer

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  - The rock carries with it thermal energy (heat)
  - The effect of advection (change in temperature) depends on the relative motion

Fig. 3.13, Stüwe, 2007
Advective heat transfer

- What happens when a parcel of rock is advected?
  - The rock carries with it thermal energy (heat)
  - The effect of advection (change in temperature) depends on the relative motion
  - The effect of advection also depends on the rate of motion

Fig. 3.13, Stüwe, 2007
Advective heat transfer

• What happens when a parcel of rock is advected?

  • The rock carries with it thermal energy (heat)
  • The effect of advection (change in temperature) depends on the relative motion
  • The effect of advection also depends on the rate of motion
  • Why?

Fig. 3.13, Stüwe, 2007
Adveective heat transfer

- What happens when a parcel of rock is advected?
  - The rock carries with it thermal energy (heat)
  - The effect of advection (change in temperature) depends on the relative motion
  - The effect of advection also depends on the rate of motion
  - Why?
  - If heat is lost to diffusion more rapidly than it is advected, advection will have little effect
Erosion

- **Erosion** results in **upward advection of rock** as surface rock is eroded and transported away.

- Upward motion brings relatively **hot rock** up from depth toward the surface, **increasing the geothermal gradient**.

- Erosion typically becomes important at **advection velocities of >0.1 mm/a**.
Sedimentation

- **Sedimentation** is essentially the opposite of erosion
- Sediment deposition results in downward advection of rock as the surface subsides and a basin is filled
- Downward motion pushes relatively cold rock downward, decreasing the geothermal gradient
- Sedimentation typically becomes important at advection velocities of >0.1 mm/a
Groundwater circulation

- Heat is also advected by flow of groundwater
  - In mountainous settings, this can become a major influence on the thermal field, depending on the rate of flow
  - The flow rate is generally controlled by the hydraulic conductivity of the bedrock

Forster and Smith, 1989
Recap

• We were introduced to the **advection equation**, related to lateral transfer/transport of a quantity.

• **Bedrock river erosion** can be simulated using the advection equation with a **spatially variable advection coefficient**.

• **Advective heat transfer** is an important heat transfer process in tectonically active regions, and its efficiency depends on the rate of advection relative to diffusion.


