Geodynamics

Lecture 11

Brittle deformation and faulting

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Goals of this lecture

• Present main ‘brittle’ deformation mechanism(s)

• Discuss the role of friction

• Relate both items above to faulting
There are five words used to describe rock deformation that are frequently misused, confused and poorly understood:

- Brittle
- Ductile
- Elastic
- Plastic
- Viscous
There are five words used to describe rock deformation that are frequently misused, confused and poorly understood:

<table>
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<th>Deformation ‘types’</th>
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</table>
Terminology, clarified

‘**Type**’ of deformation

- **Brittle**: Fracture of rock with possible slip along the fracture surface (fault); Relatively low T and P, large forces or rapid imposed deformation

- **Ductile**: Flow or coherent change in the rock in the solid crystalline state; Relatively high T and P, small forces, slow imposed deformation

Ductile deformation, Cap de Creus, Spain
Terminology, clarified

• Deformation mechanism (or law)
  • **Elastic**: Linear relationship between stress and strain; recoverable (Lectures 5-6)
  • **Plastic**: Infinite strain possible above yield stress; nonrecoverable (This lecture)
  • **Viscous**: Stress is proportional to strain rate; nonrecoverable (Lecture 12)
What was the stress-strain relationship for elastic materials?
Elasticity

\( \sigma \propto \varepsilon \)

- **Stress** is proportional to strain
- For 1-D normal stress
  \[ \sigma_{xx} = E \varepsilon_{xx} \]
  - \( E \): Young’s modulus (1D)
  - \( G \): Shear modulus (1D)
- If stress \( \to 0 \), strain \( \to 0 \) (recoverable)

Twiss and Moores, 2007
Perfectly plastic behavior

- **Constant stress** required for deformation

![Graph showing perfectly plastic behavior](image-url)

Twiss and Moores, 2007
Perfectly plastic behavior

- **Constant stress** required for deformation
- No deformation prior to exceeding yield stress
- Infinite deformation if applied stress equals (or exceeds) yield stress

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  \end{cases}
  \]
- Nonrecoverable

Twiss and Moores, 2007
What might the constitutive relationship look like for an elastic-perfectly plastic material?
Elastic-Perfectly plastic behavior

- Combination of behaviors of **elastic** and **perfectly plastic** behaviors
- Initial behavior is elastic, then plastic after reaching yield stress

Twiss and Moores, 2007
Elastic-Perfectly plastic behavior

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This combined behavior is similar to that of a fault during the earthquake cycle.
Friction in rocks

- Fault slip accounts for a large portion of deformation of the upper crust
- What must be overcome for slip to occur?
Friction in rocks

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- What must be overcome for slip to occur? **Friction**
  - After exceeding the frictional resistance, slip will occur on the fault or shear zone
  - Known as **frictional plasticity**
Friction in rocks

Fig. 8.5, Turcotte and Schubert, 2014

- Fault slip accounts for a large portion of deformation of the upper crust
- What must be overcome for slip to occur? Friction
  - After exceeding the frictional resistance, slip will occur on the fault or shear zone
  - Known as frictional plasticity
  - The basic relationship for static friction is
    \[ \tau_{fs} = f_s \sigma_n \] (Amonton’s law)
    where \( f_s \) is the coefficient of static friction, and \( \tau_{fs} \) is the static frictional stress required for slip

Normal stress
\[ \sigma_n = \frac{mg \cos \theta}{A} \]

Shear stress
\[ \sigma_s = \frac{mg \sin \theta}{A} \]
Friction in rocks

A typical value for the coefficient of static friction in rock is $f_s = 0.85$

Assuming this value, at what angle $\theta$ would the block at the left begin to slip?

You can assume that $\tau_{fs} = \sigma_s$, and recall: $\tau_{fs} = f_s \sigma_n$

Normal stress $\sigma_n = \frac{mg \cos \theta}{A}$

Shear stress $\sigma_s = \frac{mg \sin \theta}{A}$
Friction in rocks

- Upper crustal rocks generally behave as elastic-perfect plastic.
Friction in rocks

- Upper crustal rocks generally behave as elastic-perfect plastic
- For frictional slip, rock property measurements suggest the shear stress required for fault slip increases with normal force in two ‘domains’
- These are known as Byerlee’s laws

\[
\begin{align*}
\sigma_s^c &= 0.85\sigma_n \text{ [MPa]} \\
\sigma_s^c &= 50 + 0.6\sigma_n \text{ [MPa]}
\end{align*}
\]

for \( 5 \text{ MPa} < \sigma_n \leq 200 \text{ MPa} \)

for \( \sigma_n \geq 200 \text{ MPa} \)
Mohr-Coulomb criterion

- Amonton’s law, as we saw it, does not account for **rock cohesion**

  \[ \tau_{fs} = f_s \sigma_n \]

- Including **cohesion** \( c \) we can modify Amonton’s law to

  \[ \tau_{fs} = c + f_s \sigma_n \]

- This is known as the **Coulomb criterion**
Mohr found an elegant graphical representation of the Coulomb criterion that illustrates numerous items of interest, including:

- The failure envelope, cohesion and internal angle of friction
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- The **failure envelope**, cohesion and internal angle of friction

\[
f_s = \tan \phi
\]

\[
\tau_{fs} = c + f_s \sigma_n
\]

**Fig. 5.6, Stüwe, 2007**
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- The failure envelope, **cohesion** and internal angle of friction
Mohr found an elegant graphical representation of the Coulomb criterion that illustrates numerous items of interest, including:

- The failure envelope, cohesion and internal angle of friction.
Mohr-Coulomb criterion

- Plotting the state of stress of a rock as a circle with a diameter of \((\sigma_1 - \sigma_3)\), failure will occur if/when the circle intersects the failure envelope.

- In this case, failure occurs at critical shear stress \(\tau_{fs}\).
Mohr-Coulomb criterion

- The Mohr circle also portrays several other important relationships.

- The angle $\theta$ is the angle between any considered plane in a rock and the principal stress directions, given by

$$\sin(2\theta) = \frac{2\sigma_s}{(\sigma_1 - \sigma_3)}$$
Mohr-Coulomb criterion

- At what angle $\theta$ is the shear stress largest?

Fig. 5.6, Stüwe, 2007

Critical shear stress (failure)

$f_s = \tan \phi$

$\tau_{fs} = c + f_s \sigma_n$

$\sigma_S^\text{max}$

$\sigma_S$

$\sigma_1$

$\sigma_n$

$0$

$\sigma_3$

$2\theta$

$(\sigma_1 - \sigma_3) / 2$

$(\sigma_1 + \sigma_3) / 2$
The Mohr circle also portrays several other important relationships.

You can also easily see the maximum allowable shear stress in the rock and the maximum and minimum normal stresses:

\[ \sigma_{s,\text{max}} = \frac{\sigma_1 - \sigma_3}{2} \]
• Lastly, if pore pressure $p_w$ is considered, the Mohr-Coulomb criterion is

$$|\tau| = c + f_s(\sigma_n - p_w)$$

which can be formulated as

$$|\tau| = c + f_s(\sigma_n - \rho_w g y)$$

assuming the pore pressure is hydrostatic ($p_w = \rho_w g y$)
Mohr-Coulomb criterion

[Diagram showing Mohr-Coulomb criterion]

- So, what is the effect of pore fluids?
- How might they affect fault slip?

\[ |\tau| = c + f_s (\sigma_n - \rho w g y) \]
Faulting

Beartooth Plateau, Wyoming, USA
Faulting

Beartooth Plateau, Wyoming, USA
Anderson’s theory

- **Anderson** formulated the Mohr-Coulomb criterion in terms of principal stresses and lithostatic pressure \( (\sigma_{yy} = \rho gy) \)

- Anderson assumed \( \sigma_{yy} = \sigma_3 \) for a reverse fault, \( \sigma_{yy} = \sigma_1 \) for a normal fault and \( \sigma_{yy} = \sigma_2 = (\sigma_1 + \sigma_3)/2 \) for a strike-slip fault

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Fig. 5.8, Stüwe, 2007
In terms of differential stress, Anderson’s theory is

**Reverse:** \( \sigma_d = \sigma_1 - \sigma_3 = \frac{2 (c + f_s (\sigma_{yy} - \rho_w g y))}{\sqrt{f_s^2 + 1 - f_s}} \)

**Normal:** \( \sigma_d = \sigma_1 - \sigma_3 = \frac{-2 (c - f_s (\sigma_{yy} - \rho_w g y))}{\sqrt{f_s^2 + 1 + f_s}} \)

**Strike-slip:** \( \sigma_d = \sigma_1 - \sigma_3 = \frac{2 (c + f_s (\sigma_{yy} - \rho_w g y))}{\sqrt{f_s^2 + 1}} \)
Predicting fault orientation

- Now we’ll look at how to apply Anderson’s theory to predict the dip angle $\beta$ of normal and reverse faults.

- Assume principal stresses

\[ \sigma_{yy} = \rho gy \]
\[ \sigma_{xx} = \rho gy + \Delta \sigma_{xx} \]

where $\Delta \sigma_{xx}$ is the tectonic deviatoric stress, which is positive for a reverse fault and negative for a normal fault.
Predicting fault orientation

- We first need to relate $\sigma_{xx}$ and $\sigma_{yy}$ to $\sigma_n$ and $\sigma_s$ in order to apply Amonton’s law by using the equation for the normal and shear stresses in a coordinate system rotated by angle $\theta$ with respect to the principal stresses (see Lecture 3)

$$
\sigma_n = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \cos 2\theta
$$

$$
\sigma_s = -\frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \sin 2\theta
$$

- Note that here, $\theta$ is with respect to vertical, $\theta = \pi/2 - \beta$
Predicting fault orientation

If we plug in the values for $\sigma_{xx}$ and $\sigma_{yy}$, we find

$$\sigma_n = \rho gy + \frac{\Delta \sigma_{xx}}{2} (1 + \cos 2\theta)$$

$$\sigma_s = -\frac{\Delta \sigma_{xx}}{2} \sin 2\theta$$

Inserting the values above into the form of Amonton’s law that includes pore fluid pressure, $|\tau| = f_s (\sigma_n - p_w)$, yields

$$\pm \frac{\Delta \sigma_{xx}}{2} \sin 2\theta = f_s \left[ \rho gy - p_w + \frac{\Delta \sigma_{xx}}{2} (1 + \cos 2\theta) \right]$$

Note that the upper sign is for reverse faults ($\Delta \sigma_{xx} > 0$) and the lower for normal faults ($\Delta \sigma_{xx} < 0$).
Predicting fault orientation

The previous expression can be rearranged to solve for $\Delta\sigma_{xx}$

$$\Delta\sigma_{xx} = \frac{2f_s(\rho gy - p_w)}{\pm \sin 2\theta - f_s(1 + \cos 2\theta)}$$

If we assume that faulting will occur with the minimum tectonic stress, then $|\Delta\sigma_{xx}|$ should be minimized.

By setting $d\Delta\sigma_{xx}/d\theta = 0$, we find

$$\tan 2\theta = \mp \frac{1}{f_s} \quad \text{or} \quad \tan 2\beta = \pm \frac{1}{f_s}$$

where the upper sign again applies to reverse faults and the lower to normal faults.
Predicting fault orientation

Finally, the two equations from the previous slide can be combined to yield the tectonic stresses corresponding to angle \( \theta \) or \( \beta \)

\[
\Delta \sigma_{xx} = \frac{\pm f_s (\rho g y - p_w)}{(1 + f_s^2)^{1/2}} \pm f_s
\]

where the upper sign again corresponds to reverse faults and the lower to normal faults.

Figs. 8.9, 8.10, Turcotte and Schubert, 2014

Dip angle

Tectonic stress

\( \rho = 2700 \, \text{kg m}^{-3} \)

\( \rho_w = \rho_W g y \)

\( \rho_w = 1000 \, \text{kg m}^{-3} \)

\( y = 5 \, \text{km} \)
Recap

- Rocks in the ‘brittle’ domain deform elastically until their yield stress is reached.
- Once yield is reached, rocks exhibit plastic behavior, where infinite deformation is possible at a constant stress.
- Using basic geometry, fault orientations can be predicted from principal stresses using Anderson’s theory.